DOCUMENT RESUME

ED 385 439 SE 056 581

AUTHOR Phillips, Eileen; Crespo, Sandra
TITLE Math Penpals! Developing Written Communication in Mathematics.
PUB DATE Apr 95
NOTE 18p.; Paper presented at the Annual Meeting of the American Educational Research Association (San Francisco, CA, April 18-22, 1995).
PUB TYPE Guides - Classroom Use - Teaching Guides (For Teacher) (052) -- Speeches/Conference Papers (150) -- Reports - Research/Technical (143)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS Communication (Thought Transfer); *Elementary School Students; Foreign Countries; Grade 4; Higher Education; Intermediate Grades; *Letters (Correspondence); *Mathematics Instruction; *Student Evaluation; Teaching Methods; *Writing Across the Curriculum
IDENTIFIERS *Teacher Candidates

ABSTRACT

Communication in the mathematics classroom is currently viewed as an integral part of mathematics instruction. This paper explores the usefulness of math penpal letters as a means of increasing the quality of written communication in mathematics. The focus of this study is the letter writing exchanges between a class of 4th-grade students (n=28) and preservice teachers (n=27) attending a mathematics education course. The letter exchanges occurred over a period of three months, and the writing was examined for its content and style. It also served as a window through which teachers could glean some otherwise inaccessible information about students. Explorations into writing math penpal letters attest to their richness and potential for developing written communication in mathematics. The penpal exchanges also helped preservice teachers think about mathematics from the students' perspective. The study suggests that they can be a valuable addition to existing instructional practices in the mathematics classroom. Contains 10 references. (MKR)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED 385 439

MATH PENPALS!

PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
Sandra Crespo

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)
This document has been reproduced as received from the person or organization originating it.
☐ Minor changes have been made to improve reproduction quality.
• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

Developing Written Communication in Mathematics

Eileen Phillips and Sandra Crespo
University of British Columbia

Round Table Discussion, AERA Annual Meeting, San Francisco, April 1995

BEST COPY AVAILABLE

CE035581

Math Penpals!

Developing Written Communication in Mathematics

ABSTRACT: In this paper we explore the usefulness of Math Penpal Letters as a means of increasing the quality of written communication in mathematics. The focus of this study is the letter writing exchanged between a class of 28 Grade four students (nine and ten year olds) and preservice teachers attending a mathematics education course. The letter exchanges occurred over a period of three months and the writing is examined for its content and style. It is also looked at as a window through which we, the teachers, can glean some otherwise unavailable information about our students. Our explorations into writing math penpal letters attest to their richness and potential for developing written communication in mathematics. They are a valuable addition to current efforts to promote mathematical communication in the classroom.

For questions, comments, and ideas you may contact us at:

E-mail
eileen@cs.ubc.ca
smcrespo@unixg.ubc.ca

Address

Eileen Phillips
Kerrisdale Elementary School
5555 Carnarvon Street
Vancouver, B.C., Canada
V6N 1J2

Sandra Crespo
University of British Columbia
Faculty of Education
Department of Curriculum Studies
2125 Main Mall
Vancouver, B.C., Canada
V6T 1Z4

Acknowledgments
We would like to thank *David Pimm* for "listening mathematically" and contributing to the development of this project.

We would also like to thank the principal, parents, and students of Kerrisdale Elementary and UBC preservice teachers for their support and participation.

Our thanks to *Scott Higgs* for his creative cover illustration.

Why Written Communication in Mathematics?

Communication in the mathematics classroom is currently viewed as an integral part of mathematics instruction. Its importance is highlighted in the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). This document stresses the need for students to use both oral and written language to describe, explain, and justify their mathematical ideas to themselves and to others. These abilities are considered important "because they help children clarify their thinking and sharpen their understanding of concepts and procedures" (Catchcart, Pothier, & Vance, 1994, p. 33). Furthermore, it is argued that in the process of communicating their ideas, students construct, refine, and consolidate their mathematical understandings (NCTM, 1989).

A number of strategies have been proposed to encourage students to read, write and discuss ideas related to mathematics (i.e.: journal writing, small group discussions, debates). Among all forms, written communication is of special importance because it provides students with a record of their own thinking and developing ideas. Writing, as Pimm (1987) says, "externalizes thinking even more than speech by demanding a more accurate expression of ideas." (p. 115). Some researchers have also noted that the process of writing in the mathematics classroom promotes students' active involvement and it enhances their dispositions and personal construction of the subject (Borasi & Rose, 1989; Connolly, 1989; Countryman, 1992).

Writing in the mathematics classroom is seen as a powerful teaching strategy because it ensures that all students engage in the activity and not just a dominant few. It provides opportunities for those who do not feel comfortable articulating their ideas orally, to express their understanding "in a less public forum" (NCTM, 1989, p. 28). Furthermore, students' writing provide teachers with insights into their students' cognitive and affective worlds.

Why We Chose Penpals

As the popularity of writing in mathematics increases, it is important to note that the nature of the writing tasks as well as the intended audiences influence in part what and how students choose to communicate. The contexts and nature of most mathematics writing activities suggested in the literature, however, are often contrived and have the teacher as the intended audience. The purpose and the audience of students' writing undoubtedly affect the depth and clarity with which students write.

Some researchers suggest that writing activities to audiences other than the teacher are more valuable and interesting to students (Pearce & Davison, 1988) and that students generate more writing when addressing audiences other than the teacher (Miller & England, 1989). In short, it is important to recognize that learning to communicate mathematically is best accomplished in contexts where there is an authentic need to communicate (NCTM, 1989).

Often, when asked to write in journals, students fail to see a real reason for writing. It is not uncommon to hear students say, "But, I can just tell you. It's easier." Young students, in particular, don't value writing as a way of recording their thoughts and ideas because they have never been in the situation where forgetting was a problem. They either ask for the situation to be re-created, thereby allowing them the opportunity to reconstruct their thinking, or they ignore what they have forgotten as unimportant. Writing in math is an area where most students of Grade 4 age have little or no experience. It can be challenging to encourage them to see the reason for verbally explaining their thinking when they have a "right answer", let alone motivate them to write it down.

Penpal letters, we thought, would help our students (Grade 4s and preservice teachers) engage in the process of writing to learn mathematics. We were intrigued about the idea of math penpals after reading an article by Fennell (1991) about the use of math penpal letters as diagnostic tools. We wondered if math letters would increase our students' awareness of the possibilities of writing in mathematics. Penpal letters provided the younger students with a meaningful and genuine reason to communicate their ideas about mathematics. They also provided elementary preservice teachers with the opportunity to communicate mathematically, many of them for the first time, and to get a glimpse at a child's dispositions and understandings of mathematics.

Creating a Need: The Context for Writing

The math penpal project took place over a period of 10 weeks from mid-January to the end of March, 1994. The project involved 28 Grade 4 students and 27 volunteer preservice elementary teachers attending a mathematics methods course. Penpals were randomly paired. Because of numbers, one preservice teacher corresponded with two children. For the fourth graders, writing their math penpal letters was an integral part of their mathematics classes. For preservice teachers, however, writing to their math penpals was an extra curricular, non-credit, activity.

The Grade 4 students wrote six exchanges to their penpals. Four of these were letters where the topics, format, and activities were entirely up to the students; two of these "letters" were more directed. These two directed writings, however, were still open as far as format and choice of activities were concerned, but the topics were required. The university students wrote four response letters. Letters were collected and delivered to or from the school and university weekly. The letter exchange culminated in a penpal meeting hosted by the fourth graders in which penpals met and worked on various mathematics activities together. We then asked both groups of students to respond to an informal evaluation survey about their experiences as math penpals.

To introduce the idea of penpaling to her Grade 4 class, Eileen told them that a friend of hers was teaching a course on "how to teach math" to university students who were studying to become teachers. These preservice teachers, she said, were very keen to find out about elementary students' feelings and ideas about mathematics. She also added that these university students wanted to know about the kind of math that was being done in their classroom, whether it was easy or hard, whether it was fun or not, and about how to teach math effectively. Eileen suggested to her class that they might be asked to explain how they solved a math question or a problem, so that their penpal would be able to "see how they think." She also hinted that they might want to use their penpal as a "sort of private tutor," and that they would be able to ask their penpal for help with math that they found confusing or frustrating. She explained that they might be able to give their penpal advice on activities, on strategies, and on what made a "good" math program.

Sandra told her university students about the project and opened the invitation for them to participate. A letter outlining Eileen's expectations, which included meeting deadlines, the inclusion of a "fun" activity each letter, and the hope of empathetic responses when and if necessary, was given to each volunteer. This letter also expressed our belief that this exchange would be a very rewarding experience for them, both personally and professionally. Twenty-seven of the 38 students in Sandra's class volunteered.

The Grade 4 students initiated the exchange by writing their first letter to an unknown, "Dear Penpal," after first brainstorming what the content of their letter might include:

-a greeting
-tell about yourself
-tell what you like about math

-tell what you don't like in math
-write about something you want to learn about in math
-give an example of something you feel is "hard" math
-include a math problem that is fun and challenging for your penpal to try. (Make sure you can do it yourself.)
- a reminder to "sign your name."

Just prior to the end of the university term, we arranged a penpal meeting. Rather than attending class at U.B.C. the adult penpals visited the Grade 4 classroom. It was fantastic to watch the Grade 4 students lead their penpals in activities. They identified their penpals and immediately picked up where their letters had stopped. They explained math activities, played math games, asked about things that they hadn't understood from the letters, and found time just to discuss things they had in common. A light snack was provided just to nicely round the event out. For all of us involved in the project, this was one of the highlights, and we would strongly recommend that anyone trying this, or a similar activity, arrange a meeting of participants.

Insights into the Nature of Penpal Communication

The letters have proven to be an incredibly rich source of information on the types of activities that our students value as mathematics. Through the letters we learned how students who are not very verbal, either because of shyness, having English as a second language, or learning difficulties, felt about math. We also learned how persistent students could be. There were some exchanges where the same "problem" was continued back and forth until the end of the exchange period.

We believe students, both in fourth grade and in university, learned the value of writing clear explanations and learned to appreciate how much effort it takes to create a personal problem, game, or activity that someone else can work on. They learned a little bit about each other's lives and outlooks. They also learned that math problems can be started and left, and then picked up again. Not all problems had to be solved in one sitting. They learned that math writing is a special type of writing, and that it sometimes helps to include examples and diagrams.

We will share some of our insights into our students' penpal exchanges by highlighting some interesting patterns and themes we have observed. We will share the excerpts selected with the following framework in mind. First, we look at how the role of advisor inspired the younger students to improve their writing and encouraged them to lead the mathematical discussion. Then we will illustrate how preservice teachers'

writing served as models to the younger students who seemed to mirror their older penpals' style, form, and tone of written communication.

Please note that we have corrected spelling in the exchanges quoted because misspellings can be distracting, but we have left the grammatical structure as written.

Grade 4 Students as Advisors and Initiators of Topics

Eileen had encouraged her students to provide advice to their preservice teachers' penpals. They were also encouraged by their adult penpals who were eager to listen to their opinions and feedback on the problems and questions they sent in their letters.

> «I am studying math, but from the application side of things (how to teach math concepts). This includes finding interesting and innovative ways of teaching math. Any suggestions that you have in making math more fun would be appreciated» [K/] 1st letter, adult penpal]

> «I am doing an oral presentation on problem solving soon. Could you give me an example of the problems you are working on? This would really help me to see what grade fours are doing in math» [A/D 1st letter, adult penpal]

The Grade 4 students took their role as advisor quite seriously. Some seemed to take on the responsibility of reporting the types of activities they had worked on every week, and some others provided preservice teachers with advice, quite often unsolicited. Topics ranged from the use of calculators to working in groups. For example:

> «This week in math we did some times tables. We did estimating by rounding off. The times tables are getting harder and harder because we multiply by higher numbers. I think that rounding off is easy because it is just a review of it. The other thing we did this week is do a color map. What we did was got a paper with spaces to color in. In the older days crayons were very expensive so the point was to use the least amount of color as possible. The rule was that you can't put two of the same colors beside each other. The lowest amount of colors that we could get was three colors.» [J/K 2nd letter, child penpal]

> «The next point I want to make is let kids work with other kids when they do math because it helps them get the sense of the problems» [B/S 2nd letter, child penpal]

> «It is good to be fun in math. I think it is boring to just do textbook math. It is good to do puzzles and figure out patterns and break the code. I think it is [good] to do textbook and some math games and other stuff» [N/V 1st letter, child penpal]

On other occasions Grade 4 students provided preservice teachers with a different way to interpret or to look at mathematical questions and problems. In the following exchange the fourth grader gets to point out to her adult penpal that sometimes mathematics does not necessarily get you the "real" answer.

Grade 4: If you feed a baby 8 ounces of milk in one feeding how many ounces would it be in two feedings?

Adult penpal: The second question was interesting since you are expecting a baby girl or boy into your family soon. The answer is:

8 oz. **x** **2** **=** **16 oz. for 2 feedings.**
per feeding # of feedings

Grade 4: You were close for the answer of my question (about the baby) but the right answer is 15 oz. or less because the baby will either throw up or the milk will dribble out the sides. I read this in a magazine.

This is an example from the first letter exchange. It is interesting to note that this pair established personal rapport very quickly. The Grade 4 had mentioned a new baby on the way, and the preservice student commented that this made the problem very relevant. Also, the preservice student answered the question by clearly explaining how the answer was achieved. In reply to the answer, the Grade 4 student next explained that the stated answer was "close", thereby demonstrating some empathy with the penpal. The "right" answer was then very clearly stated.

This Grade 4's choice of question is interesting for two reasons. First, it had an approximate solution. This is important because it shows that mathematical ideas translated into "real world" situations often are not precise. Second, the information source was a magazine, not the textbook or the classroom. The example indicates this Grade 4's awareness that math exists outside school and her ability to create math problems from non-traditional sources.

Grade 4 students seemed to take prime responsibility for selecting the content topics discussed in the letters. Perhaps this was because the Grade 4 wrote the initial letter and the preservice teacher naturally kept to the initiated topics as a source of response material. We noticed that when preservice teachers acknowledged, followed, and/or extended the topics introduced by their penpal, the Grade 4 penpals' responses were more detailed and richer than otherwise. We also noticed that in such cases, the pair engaged in what Pirie and Schwarzenberger (1988) call "genuine mathematical discussion." There was a sense of purposeful written conversation between the two penpals as their discussions moved forward with each party building on the other's ideas. The following exchanges help illustrate this point.

Grade 4: We have been talking about the names of shapes. What is this shape ?

Adult penpal: I think this shape is called parallelogram. I have an activity for you to try. In the envelope there are some toothpicks. See how many triangles you can make from them.

Grade 4: I figured out the math question, I made five triangles with toothpicks. [At the end of letter penpal wrote] P.S. This is what I did with the toothpicks

What shape is this □?

Adult penpal: Thank you for drawing your toothpick triangles. You made 5 separate triangles. What is the highest number of triangles you can make when you join the triangles together? Hint:

This toothpick is the side for 2 triangles

Grade 4: When I joined the tooth picks together I made eight triangles. I had one tooth pick left over. This is what it looks like:

How many trapezoids and parallelograms can you find?

Adult Penpal: The triangle shape you made is very good. I found 5 parallelograms. I forget what a trapezoid is. Could you remind me?

On the other hand, when the preservice teacher did not maintain the role of topic-follower, then some interesting tactics were tried by many of the Grade 4's to restore what, we believe, they perceived was the "right" order. From looking through the letters, it seems that when an adult penpal did not continue an activity, but chose to initiate his/her own direction, that the Grade 4 penpal usually did not respond to the change. Rather, he/she either continued presenting the original topic or changed to a new topic of his/her own choosing.

Grade 4: Can you make this shape using pattern blocks? Use six shapes. [This activity is repeated three times, with three different large drawings that are really outline tracings of the blocks used.]

Adult penpal: [No reference made to the above shape activity.] Here is a puzzle for you to solve. Place 17 sticks so that they will form a pattern of 6 squares ...

Grade 4: Do you know what pattern blocks are? I hope you do. [Nothing mentioned about the stick puzzle.]

Adult penpal: I hope you had time to solve the puzzle I sent you last time. Here is the answer. [No mention of pattern blocks.]

Grade 4: Can you make these shapes using pattern blocks? [Gives a series of six shapes.]

Adult penpal: Your last letter was wonderful - you are very creative with making patterns and posters.

This last comment was the closest reference to the "pattern blocks activity" in these exchanges. When we first looked at the letters of this pairing, we were struck that they

never really got to each other's math. A lot of their dialogue was about families (brothers and sisters). We think now that this was because the family topic originated with the Grade 4 student and was continued by the adult penpal. Whereas the math, although continued as a general subject, was never reciprocated by same topic/ activity writing, that is, the area chosen by the Grade 4 penpal was never acknowledged. Re-reading these exchanges we are struck by the persistence of the Grade 4 student. Perhaps this math attitude of "sticking with something" would have paid off if there had been a few more exchanges. It seems as if the adult penpal started to recognize that her partner's drawings had some significance.

Preservice Teachers as Models of Writing and Tone

We also observed that the clarity, depth, and amount of writing of the Grade 4 students tended to mirror the writing of preservice teachers. For example one Grade 4 student's writing developed from "The riding problem I figured it out on a piece of paper it was really fun but easy. The answer is 7 saddles" to more embellished articulations of the solution process, such as:

> «I really like your problems and I loved the card [Valentine's] with the math problem because we are just learning about divs. This is how I did the problems. [Shows all the work, for the Valentine's problem and she adds] It was a little confusing at first. I decided to use the word split as share. I used every heart in the picture. I wasn't sure if I was supposed to. I had to act the problem out with friends. You would have to give 7 hearts each and 3 left over.» [M/A 3rd letter]

The mirroring of phrases and explanations can be appreciated in the following pair's exchanges. Notice the use of "the answer to your first question is" being reflected back as well as the reference to "tricky questions." The expression of doubt as to the correctness of the solution was also used by the Grade 4 penpal.

Adult penpal: The answer to your first question is:
510 - 520 - 500 - 530 -490 - 540 - 480 - 550 - 470 - 560 ...
This question was a little bit tricky at first. The second question was interesting ...

In another letter ...

Adult penpal: It took me a while to figure out your math puzzle, but I finally got the pieces together and came up with these answers [lists] -3.4, 60.504, 2003.1, 1.05. I hope these are right, but if not be sure to let me know. Your riddle stumped me. My only guess is that somehow it remained scoreless (0 - 0), maybe the refs. didn't show up or something. Please let me know the answer to this.

In the fourth letter...

Grade 4: Thanks for the letter. You really put in some tricky questions. The answer to your first question is 31 x 32 x 33 = 32, 736 what I did to get the answer was I tried in the 50's and it was too high so I kept trying 'til I got it. Your second question looks so easy. Here's the answer [shows work]. The answer to your last question could be I'm not sure but I think

is 20 lizards. The question with the dots was tricky and I'm still working on it and I'll give you the answer on Friday [Penpal Visit Day].

In a few of the penpal pairings a formal style of writing was demonstrated by the adult penpal. The Grade 4 penpal generally did his/her best to follow the tone set by the preservice teacher penpal. For example:

Adult penpal: Did you know there are logical steps one follows in order to get the answers to problems? Did you also know that as teachers, we are not so concerned about the product (answer), but rather we focus more on the process (how you derived the answer).

Grade 4 : The last thing we did was learn Algorithm for times tables (the traditional way for doing times tables).

Notice how the style of using parentheses for explaining terms has been adopted. Also, in another part of the reply, in the same letter exchange as above, there is a copying of tone and words.

Adult penpal: Well, you stumped me! Although it wasn't the problems that confused me, the Lattice Multiplication did.

Grade 4: I'm not surprised that the Lattice Multiplication stumped you. When I first learned it, the method was really confusing.

The tone of the Grade 4 response sounds a little more "mature" than is normal for students of this age. Notice the word-copying of "stumped" and "confused/confusing." It also appears that it is form, rather than content, that is copied by the younger student.

Following is another pair's exchange that exemplifies the above setting of tone by the adult penpal and setting of content by the Grade 4 partner.

Grade 4: Our math so far has been quite easy and I like most of it. I like it because it is interesting and fun.

Adult penpal: You mentioned in your letter that you find math interesting and fun. What is it that you find fun and interesting in math? Maybe in your next letter you can fill me in?

Grade 4: You asked me to tell you why I think math is interesting. Well, when we get homework, I do all the questions I can do by myself. If I see any that are tricky, I take them to school the next day. When we mark them, I get the answer and once you get it, it becomes more easy. That's what makes it interesting. That also makes it fun.

Adult penpal: I think that it's great that the questions that you find difficult for homework you do not give up on. Sometimes when one discusses the homework the next day the answers might come easier.

The tone of the above exchange is very relaxed, and the explanations are complete. Notice the copying style of, "You asked me to tell you ..." with, "You mentioned in your letter ..." The topic of conversation, however, remained in the hand of the Grade 4 penpal.

Further Insights and Spin-Offs

Math penpal letters are, we believe, rich sources of information. They have provided us with insights into the advisory role and mirroring styles as we have discussed in the previous section. They have also provided us with information about our students' attitudes, inferences, social learning, and metacognition. We would now like to briefly discuss these additional benefits of letter writing.

Perceived Insights into the Grade 4's Affective Domain

Much of the writing in these letters offered a window into students' beliefs, feelings, and thinking. They wrote about what they liked, didn't like, and what they wanted to learn more about. Grade 4 examples include:

> «I would like to learn how to do division. I like plus the best but I do not like times as much as plus.»

This student has shown, through her language, that she relates to math operations as activities, complete in themselves, that can be compared like preferences for chocolate or nuts. She gave no hint of needing a context for her opinions.

"Fun" was a theme that echoed throughout the letters of the students who said they liked math. Reading this over and over confirms the connection between pleasure and learning. Fun, in math, is not used to connote superficial enjoyment, but usually implies engagement in a topic or task. Another theme that ran throughout the affective types of comments was that games are valued in math as a source of enjoyment. Often what a student "likes" in math are the activities such as games and problems solving situations. Even students who said that they didn't really like math, gave games and/or problem solving as something that they do like.

> «Math is one of my favourite subjects. My favourite subject in math is problem solving.»
>
> «I like playing card games in math. I also like problem solving and place value.»
>
> «I like playing place value games with dice.»
>
> «I'm not crazy about math but I like magic squares and the King's Table.»
>
> «The only thing I like in math is games.»

We have referred to the benefits of having a strong writing rapport between penpals. It seems that there are unwritten/unstated "rules" operating that require the adult penpal to stay on the topics generated by the Grade 4 penpal, unless the topic change is a part of a game activity that holds appeal for the younger student. In other

words, the body of the letters seems to be guided by the Grade 4 partner, whereas an investigation activity can be posed by either partner. However, this type of activity will only be reported back on if it has appealed to the recipient.

We were amazed at the perseverance shown by many of the students. Often their interest was maintained into the next letter exchange. One Grade 4 wrote, "The question with the dots was tricky and I'm still working on it." Another explained, "I thought it [the letter] was interesting but the problems were a little too hard. I can't wait until the next letter to get the hints and try again." In his next letter he continued, "I still can't get all of the problems you gave me in your first letter." This particular problem was not resolved until the day of the penpal meeting. We saw these two penpals working at the board making up palindromes, the topic that had been under discussion for the last three exchanges!

Often the form of an activity would be picked up and adapted by the penpals. There are examples to show that this type of extension happened with activities that originated with the Grade 4 and with activities that originated with the adult penpal. For instance, a treasure hunt grid, using coordinates to find the route to the treasure, originated with one of the adult penpals. This grid format was adapted by the Grade 4 penpal recipient to show the order in which you tack up a horse. Another Grade 4 student, a friend of the first, also adopted this activity in a problem asking her penpal to "find which horse to ride."

Another example that we'd like to share resulted when the Grade 4 students wrote to their penpals about the use of calculators. They wrote their opinions in the form of an acrostic poem, a list of ideas, a web, or a paragraph. In one exchange, the adult penpal responded to her penpal's acrostic with her own ideas written in acrostic format too. Soon, many of the Grade 4 students were aware of this poem and wanted to see/read it.

In the above cases of "networking," and in the other instances of it that occurred, the swiftness and the silence of the network chain was amazing. Adaptations of problems would be noticed in the Grade 4 letters, and only then would we be aware that inter-class communication had resulted in sharing. Sometimes it was possible to see someone working on a solution and later realize that the student was working on one of his friend's problems. This really impressed us because independent learning that guides and sustains itself is unusual in children of this age, at least in the school setting. This is more like the type of involvement that can be seen in hobbies or sports or "free-

time" explorations. This truly is exciting because it exemplifies the kind of life-long learning style that many educators are trying to promote.

Perceived Pedagogical Benefits for Preservice Teachers

The penpal experience proved to be quite successful in terms of the interest and enthusiasm pupils and preservice teachers brought to the activity. For some preservice teachers this activity was more enjoyable and rewarding than their actual course assignments. In their letters, some preservice teachers told their penpals how much more they enjoyed their letter writing than their actual course work. One preservice teacher even claimed that "I enjoyed the time I took to write the letters, because it allowed me to get away from assignments and projects."

The penpal exchange also provided preservice teachers with a context for trying out, putting into practice, exploring, and experimenting with ideas they were learning during their methods course. Many of the mathematical tasks and problems preservice teachers posed in their letters were selected or adapted from classroom activities and readings discussed during class. In addition, some preservice teachers indicated that this experience helped them "learn things about children and how they relate to mathematics which I probably would not have seen until my full term practicum." In addition, most preservice teachers indicated that "being able to see math from a grade 4 students' perspective" was very beneficial to them as future teachers.

For some preservice teachers, the experience also served to challenge some of their preconceived notions about mathematics, the teaching and learning of mathematics, and of students' cognitive and affective dispositions related to mathematics. The following excerpts illustrate how the pupils' writing about their experiences in their mathematics classroom helped preservice teachers reconsider some of their previous ideas about mathematics and the mathematics classroom.

> «Your letter about calculators was very interesting. Before your letter about calculators I did not think that calculators should be allowed in the classroom. I now realize that calculators can be very helpful with checking answers and word problems» [D/T 3rd letter, adult penpal].

> «Before I started this Teacher Education program I thought that math was mostly doing questions out of a textbook, like I had when I was in elementary school. By talking to you and Sandra, I've come to realize that math is so much more. It's fun, especially doing things like King's Castle & playing math games!» [A/M 4th letter, adult penpal]

Another such opportunity arose when one child wrote "I don't really like math but I'm really good at it" and another said "I love math but I am not too good at it. I like it

because it is fun to do." Many preservice teachers expect below average students to dislike mathematics and expect positive feelings about math only from those who excel in the subject. They would not associate negative feelings towards mathematics with students who do well and would not expect students who "struggle" to actually like it. Similarly, the excerpts below served to challenge preservice teachers' notions of what makes mathematics fun. Preservice teachers often say they want to make mathematics fun and enjoyable for their students. However, not many of them would consider a hard math question or problem to qualify as enjoyable. Yet to these Grade 4 penpals, a good mathematics challenge is what makes mathematics worth doing.

> «That was a great question it took lots of thinking but it was fun» [D/S 3rd letter, child penpal].
>
> «I like word problems because they make you think a lot» [D/S 4th letter, child penpal]
>
> «I liked the problem you gave me. The first one about the horse was a bit tricky.at first ... It was fun! The second problem I found a bit harder. I liked it more than the one before.» [S/T 2nd letter, child penpal].

Another interesting spin off for preservice teachers was to have their penpals teach them some new mathematical ideas and procedures. This, no doubt, was a humbling experience for many of them – to discover that their children penpals were capable of understanding some mathematics they themselves found difficult or that they had not encountered before.

> «You did better with the pizza puzzle than I did. I was able to cut 7 pieces with only 3 cuts. When I cut the pizza with only 4 cuts I was only able to get 10 pieces, but you got 11. Congratulations!» [K/E 2nd letter, adult penpal].
>
> «Your math game was very hard, and I wasn't sure how to do it. Next time, if you give me an example, it might make it easier for me» [M/J 1st letter, adult penpal].
>
> «Until your letter, I had never seen lattice multiplication. It took me a while to figure it out, but I think that I've got it.» [M/M 3rd letter, adult penpal]
>
> «Your explanation on the lattice multiplication is very good. You wrote very good directions. I understand how to do it. I've never seen it before. I think I will teach this to my students when I am a teacher» [D/P 3rd letter, adult penpal].

Not surprisingly, some preservice teachers found that "it [penpal project] showed me that students are much more capable than I originally thought. I was very impressed with [penpal's] problem-solving skills and mathematical thinking."

The penpal exchanges also helped preservice teachers begin to think about mathematics from the students' perspective since they had to select, adapt, or construct mathematical challenges that would be interesting to their penpals. This, according to

some preservice teachers, was how they benefited from writing penpal letters. In response to the question *The best thing about penpal letters was ...* some wrote:

> «Writing to the penpal and thinking of interesting problems to ask him»
>
> «Searching for math problems that [penpal] may be interested in and able to do»

In addition to gaining insights into children's views about mathematics and their "likes and dislikes" about the subject, the penpal exchanges provided preservice teachers with a window into "what grade 4's are doing." To others, the letters "shed light on what was going on in the classroom and this for me (as a prospective teacher) took some of the fear away regarding what it is that I will ever teach in my own class." Others claimed that the personal interaction with a student allowed them to get in touch with the things that mattered and interested students at that grade level. This, some said, would help them make mathematics more meaningful and more related to their students' world since, as one preservice teacher said, "it's sometimes hard to remember what I was like 14 years ago."

Closing Note

Our observations and analysis suggest that through the penpal exchange students became more able to communicate mathematically through writing. The quality and quantity of each student's written communication increased intermittently as the project progressed. The letters provided both groups of students with a vehicle for sharing ideas, requesting information and explanations, posing questions and problems, giving and requesting advice, and extending mathematical challenges.

Moreover, the mathematical content of the letters not only reflected what the students were learning in school, it also reflected out-of-school mathematics, and the mathematics that the students were constructing through the penpal interaction. In addition, many students also demonstrated a willingness to persevere with an idea, task, question, or problem over several letter exchanges. These findings indicate that penpal letters are beneficial not only for fostering written communication in mathematics but also for the development of mathematical attitudes, self-confidence, mathematical problem solving skills, and reasoning skills.

The penpal letters provided all participants (elementary and preservice students and instructors) opportunities to reflect and inquire into their developing mathematical understandings. They offered a unique multi-directional link between elementary

students, preservice teachers, and ourselves as their teachers. Our findings indicate that math penpal letters presented a genuine context for students to write and read about mathematics in a more personal and relevant manner than regular classroom writing would normally provide. This study attests to the educational potential of letter writing as a tool for engaging students in collaborative mathematical inquiry. It also suggests penpal letters can be a valuable addition to existing instructional practices in the mathematics classroom.

References

Borasi, R. & Rose, B. J. (1989). Journal writing and mathematics instruction. *Educational Studies in Mathematics, 20*, 347-365.

Catchcart, W. G., Pothier, Y., & Vance, J. H. (1994). *Learning Mathematics in Elementary and Middle Schools*. Scarborough, Ontario: Allyn & Bacon Canada.

Connolly, P. (1989). Writing and the ecology of learning. In P. Connolly & T. Vilardi (Eds.). *Writing to Learn Mathematics and Science*. NY: Teachers College, Columbia University.

Countryman, J. (1992). *Writing to Learn Mathematics*. Portsmouth, NH: Heinemann.

Fennell, F. (1991). Diagnostic teaching, writing and mathematics. *Focus on Learning Problems in Mathematics, 13*(3), 39-50.

Miller, L. D. & England, D. A. (1989). Writing to learn algebra. *School Science and Mathematics, 89*(4), 299-312.

National Council of Teachers of Mathematics (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA.: Author.

Pearce, D. L. & Davison, D. M. (1988). Teacher use of writing in the junior high mathematics classroom. *School Science and Mathematics, 88*(1), 6-15.

Pimm, D. (1987). *Speaking Mathematically: Communication in Mathematics Classrooms*. London: Routledge.

Pirie, S. E. B. & Schwarzenberger, R. L. E. (1988). Mathematical discussion and mathematical understanding. *Educational Studies in Mathematics, 19*, 459-470.